

### 3. ATWOOD'S MACHINE

STRUCTURED

#### Driving Question | Objective

How is the acceleration of the two masses of an Atwood's machine affected by their difference in mass and by their total mass? Experimentally determine the mathematical relationship between the acceleration of an Atwood's machine, the difference between its two masses, and the sum of those two masses.

#### Materials and Equipment

- Data collection system
- PASCO Smart Gate photogate<sup>1</sup>
- PASCO Super Pulley with Mounting Rod<sup>1</sup>
- Mass and Hanger Set
- Table clamp or large base
- Support rod, 60-cm or taller
- Right angle clamp
- Thread, about 1 m
- Scissors

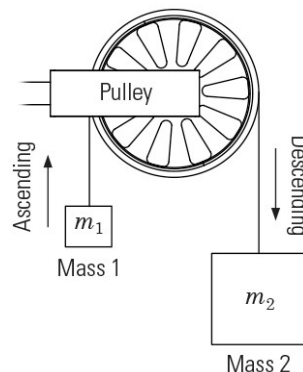
<sup>1</sup>[www.pasco.com/ap21](http://www.pasco.com/ap21)



PASCO Smart Gate

#### Background

An Atwood's machine consists of two masses connected by a light thread over a pulley. If the mass  $m_2$  in the diagram at right is greater than  $m_1$ ,  $m_2$  will accelerate downward and  $m_1$  will accelerate upward. If the string connecting the masses is taut and does not stretch, the two masses will experience the same acceleration and they can be considered to be a single system. The acceleration of this system will depend on both the mass difference between the two masses and the total mass of the system, which is the sum of the two masses.

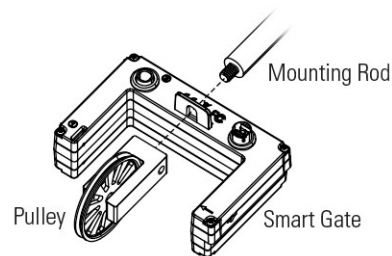


#### Procedure

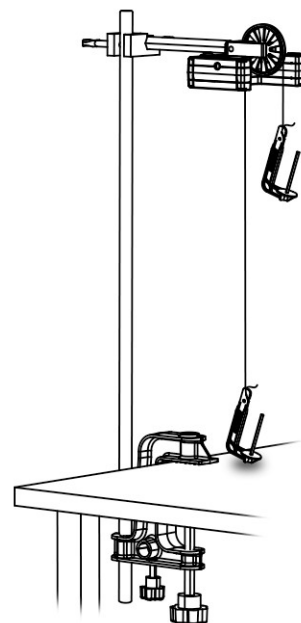
##### Part 1 – Varying Mass Difference

##### SET UP

1. Assemble and mount the table clamp and support rod on the edge of a table, or assemble the base and support rod and place it on the tabletop.
2. Attach the pulley to the tab of the photogate using the mounting rod so that the spinning pulley spokes will interrupt the beam of the photogate.



3. Attach the mounting rod horizontally on the support rod using the right angle clamp. Place it near the top of the rod with the pulley over the tabletop.
4. Cut a length of thread about 15 cm longer than the distance from the top of the pulley to the tabletop.
5. Place the thread on the pulley, threading one end through the gap between the pulley and its frame. Tie a loop on this end of the thread and place an empty mass hanger on it. Rest the attached hanger on the tabletop.
6. Tie a loop on the other end of the thread just below the pulley. Place the other empty mass hanger on this loop.
7. Connect the photogate to your data collection system.
8. Configure the data collection system for a Photogate with Pulley or Smart Pulley (Linear) with the default spoke arc length of 0.015 m to measure the linear speed.
9. Create a graph display of Linear Speed on the  $y$ -axis with Time on the  $x$ -axis.
10. Add 140 g of mass (suggested masses: 100-g + two 20-g) to the 5-g hanger resting on the tabletop, for a total of 145 g hanging from the string on the side of the pulley closer to the mounting rod.



*NOTE: Throughout this activity, the mass hanging on the side of the pulley closer to the mounting rod will be the lesser of the Atwood's machine's two masses and will be referred to as Mass 1.*

11. Support the suspended hanger with your hand to prevent it from dropping as you add the following masses to it. Add 195 g of mass (suggested masses: 100-g + 50-g + 20-g + 10-g + three 5-g) to the 5-g suspended hanger, for a total of 200 g hanging from the thread on this side. Continue to support it with your hand.

*NOTE: Throughout this activity, the mass hanging on the side of the pulley farther from the mounting rod will be the greater mass and will be referred to as Mass 2.*

12. Before you collect data, practice releasing and catching the masses: Slightly lower Mass 2 so Mass 1 lifts just off the tabletop. Once any swinging has settled, release Mass 2 and then gently catch the rising mass just before it strikes the pulley. Once you are done practicing, return Mass 1 to the tabletop and continue holding the greater mass suspended just below the pulley height.

#### COLLECT DATA

13. Begin data recording.
14. Release Mass 2 and then catch the rising Mass 1 just before it strikes the pulley.
15. Stop data recording.
16. Gently lower Mass 2 to rest on the tabletop.

*NOTE: If the two mass hangers collided, delete the run and record another.*

17. Use the tools on your data collection system to determine the slope of a linear fit to your Linear Speed versus Time data during the time when the masses were moving freely. Record this as the *acceleration* of the system in Table 1.

18. Also in Table 1, record the mass of Mass 1 (including the hanger) and that of Mass 2 (including the hanger).
19. Repeat data collection four more times, transferring 5 g from Mass 2 (the greater mass) to Mass 1 between each trial.

### **Part 2 – Varying Total Mass**

#### **COLLECT DATA**

20. Continue with the Part 1 setup for Part 2.
21. Copy the data from your final trial (Trial 5) in Part 1 into the first row of Table 2.  
*NOTE: This eliminates the need to repeat data collection for this same mass combination in Part 2.*
22. Remove 30 g from each of the mass hangers, leaving a total of 135 g (including hanger mass) on Mass 1 and 150 g (including hanger mass) on Mass 2.
23. Repeat the data collection steps from Part 1 to determine the system's acceleration. Record the acceleration in Table 2 as Trial 6. Also record the masses Mass 1 and Mass 2.
24. Repeat data collection four additional times, removing an additional 30 g from both sides between each trial.

## Data Analysis

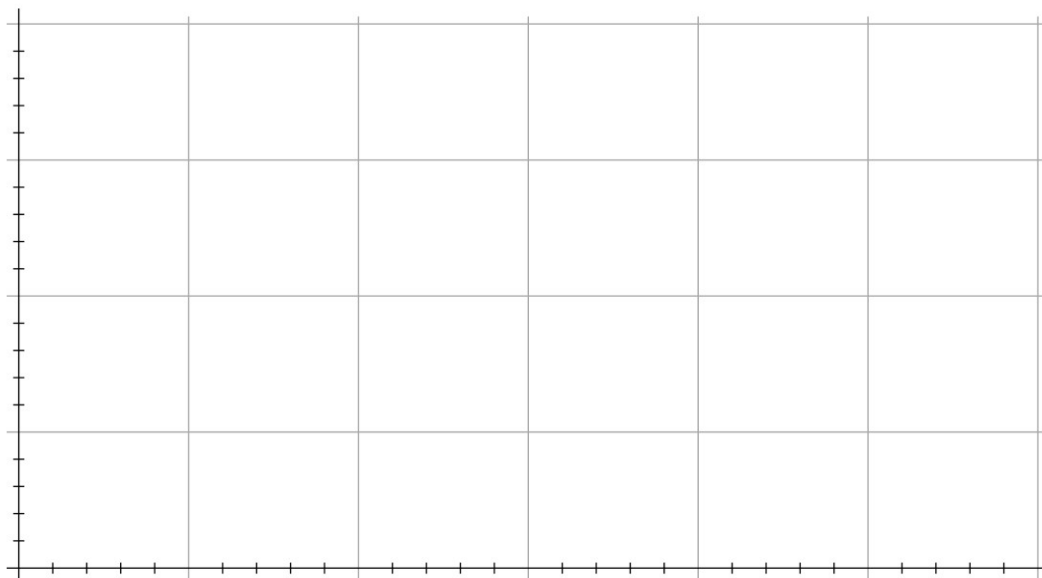
### Part 1 – Varying Mass Difference

Table 1: Varying mass difference with constant total mass

Trial	Acceleration (m/s <sup>2</sup> )	Mass 1 (kg)	Mass 2 (kg)	Mass Difference (kg)	Total Mass (kg)
1					
2					
3					
4					
5					

1. Calculate the difference between Mass 2 and Mass 1,  $m_2 - m_1$ , and total mass,  $m_2 + m_1$ , for each trial in Part 1. Record the results in Table 1 in units of kilograms (kg).
2. Plot a graph of *acceleration* versus *mass difference* in Graph 1. Be sure to label both axes with the correct scale and units.

Graph 1: Acceleration versus mass difference with constant total mass



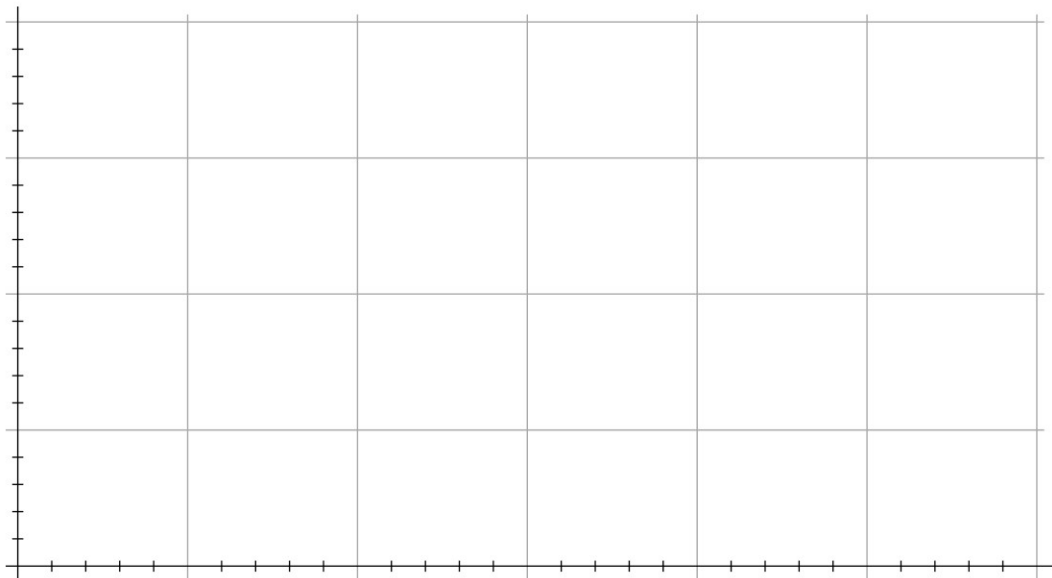
Part 2 – Varying Total Mass

Table 2: Varying total mass with constant mass difference

Trial	Acceleration (m/s <sup>2</sup> )	Mass 1 (kg)	Mass 2 (kg)	Mass Difference (kg)	Total Mass (kg)	1/Total Mass (kg <sup>-1</sup> )
5						
6						
7						
8						
9						
10						

- 3. Calculate the mass difference,  $m_2 - m_1$ , and total mass,  $m_2 + m_1$ , for each trial in Part 2. Record the results in Table 2 in units of kilograms (kg).
- 4. Calculate the inverse of the total mass (1/total mass) for each trial in Part 2. Record the results in Table 2 in units of inverse kilograms (kg<sup>-1</sup>).
- 5. Plot a graph of *acceleration* versus *1/total mass* in Graph 2. Be sure to label both axes with the correct scale and units.

Graph 2: Acceleration versus 1/total mass with constant mass difference



## Analysis Questions

- ❓ 1. For each part of your experiment, list each variable and indicate whether it was held constant, increased, or decreased.

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- ❓ 2. How did changing the difference in mass between the two sides affect the acceleration of the Atwood's machine?

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- ❓ 3. Based on your data, express the relationship between the acceleration  $a_y$  and mass difference  $m_2 - m_1$  by completing this proportionality statement:

$$a_y \propto \underline{\hspace{2cm}} \quad (\text{total mass held constant})$$

- ❓ 4. How did changing the sum of the two hanging masses affect the acceleration of the Atwood's machine?

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- ❓ 5. Based on your data, express the relationship between the acceleration  $a_y$  and total mass  $m_2 + m_1$  by completing this proportionality statement:

$$a_y \propto \underline{\hspace{2cm}} \quad (\text{mass difference held constant})$$

- ❓ 6. Combine the two relationships above into a single proportionality expressing the relationship between the Atwood's machine's acceleration  $a_y$ , the mass difference  $m_2 - m_1$  and the total mass  $m_2 + m_1$ :

$$a_y \propto \underline{\hspace{2cm}}$$

- ❓ 7. Convert the proportionality statement above into an equation by introducing a proportionality constant  $k$ :

$$a_y = \underline{\hspace{2cm}}$$

8. Use your data to determine the proportionality constant  $k$ . Be sure to specify its units. Briefly explain how you determined its value and units.

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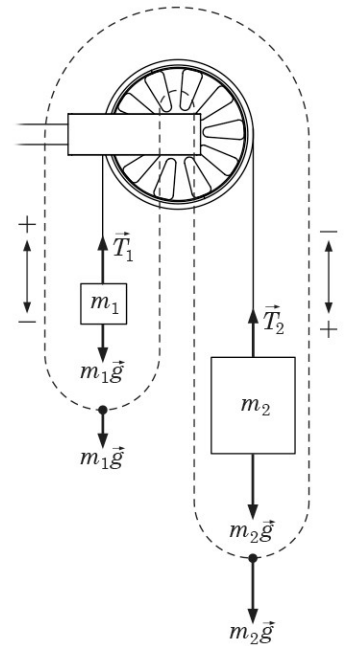
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9. Consider this free-body diagram of an Atwood's machine. Assume that the masses of the string and pulley are negligible. The analysis can be simplified if the system is defined to consist of the two masses linked together, as indicated by the dashed line. This allows you to disregard the string tension as an internal force and consider only the two forces  $m_1g$  and  $m_2g$  acting on the system. You can also consider the system to be moving in one dimension, with *positive* defined in the direction of  $m_1$  ascending and  $m_2$  descending, as indicated.

Apply Newton's Second Law,  $\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$ , to derive an expression for the acceleration  $a_y$  of the system in terms of the masses  $m_1$  and  $m_2$ .



10. How does the expression for acceleration that you determined from your data analysis compare to the equation derived above from Newton's Second Law? Justify your answer.

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**Synthesis Questions**

- ❓ 1. One way to check whether a derived relationship is reasonable is to consider whether it behaves as expected in extreme or *limiting* cases. Determine whether the relationship you derived between the acceleration  $a$  of an Atwood's machine and its two hanging masses  $m_2$  and  $m_1$  reduces to a reasonable form when the two masses are equal. Explain your reasoning.

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- ❓ 2. Similarly, determine whether the relationship you derived between the acceleration  $a$  of an Atwood's machine and its two hanging masses  $m_2$  and  $m_1$  reduces to a reasonable form when the mass  $m_2$  is much greater than  $m_1$ . Explain your reasoning.

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- ❓ 3. A planetary rover carries an Atwood's machine with 100 g on one side and 110 g on the other. If the system's acceleration is measured to be  $0.176 \text{ m/s}^2$  on a certain planet, what is the acceleration due to gravity on that planet? Which planet in the solar system is it?

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- ❓ 4. What ratio of masses  $m_2/m_1$  would produce an Atwood's machine whose acceleration is half that of an object in free fall?



5. Elevators cars have counterweights to reduce the amount of work motors need to do to lift the car. You might idealize an elevator system without its motor as an Atwood's machine. If a particular elevator's counterweight mass is 1,000 kg and its elevator car and passengers have a combined mass of 1,200 kg, what acceleration would the passengers experience if the motors and safety braking mechanisms failed? If the elevator car accelerated from rest at a height of 12 m above the ground floor, how long would it take for the car to reach the ground floor? What would be its speed on impact?