Oscillation Equations of Motion

Equipment

1	Motion Sensor	PS-2103A
1	pendulum clamp	ME-9506
1	Density Set	ME-8569
1	Spring Set	ME-8999
1	Braided String	SE-8050
1	90 cm Rod	ME-8738
1	Large Rod Base	ME-8735

Introduction

The motion of an oscillating mass is measured using a Motion Sensor, and graphs are produced of position, velocity and acceleration. The relative phase of each is examined, and compared to theory.

Calculations are made for the maximum velocity and maximum acceleration during an oscillation, and compared to values measured directly off the graphed data.

Theory

For an object oscillating in simple harmonic motion, its displacement from equilibrium, x, can be written as

$$x = A\cos(\omega t) \qquad (1)$$

where A is the initial amplitude and ω is the angular frequency. Note: For simplicity, we assume here that the object is released from rest.

To find the velocity, v, we take a derivative of Equation (1).

$$v = dx/dt = -\omega A\sin(\omega t) = \omega A\cos(\omega t + \pi/2) \qquad (2)$$

Note the use of a trig identity to write the velocity in the form of $\cos(\omega t + \varphi)$, where φ is the phase angle.

To find the acceleration, a, we take a derivative of Equation (2).

$$a = dv/dt = -\omega^2 A\sin(\omega t + \pi/2) = \omega^2 A\cos(\omega t + \pi) \qquad (3)$$

Equation (2) shows that the maximum velocity, v_{max} ,is

$$v_{max} = \omega A \qquad (4)$$

and the maximum acceleration, a_{max} , is

$$a_{max} = \omega^2 A \qquad (5)$$

Setup

1. Use the large base and 90 cm rod to support the pendulum clamp as shown in Figure 1.
2. Position the Motion Sensor as shown and plug it into the interface.
3. In PASCO Capstone, set the sample rate to 50 Hz. Create a graph of Position vs. Time. Change the units on the Position axis to cm.
4. Aim the Motion Sensor upwards, towards the pendulum clamp.
5. Choose one of the long, stronger springs from the Spring Set. The spring with the larger spring constant should have a spot of red paint on one end.
6. Tie a short piece of string through the hole in the brass mass as shown in Figure 2. Hang the mass, string, and spring assembly from one of the hooks on the back of the pendulum clamp.
7. Click on the properties (gear) icon for the Motion Sensor in the Hardware Setup window. With the mass hanging at equilibrium (not moving), click Zero Sensor Now.
8. Allow the mass to oscillate and take some practice data. Adjust the Motion Sensor, if necessary, to get smooth continuous data.

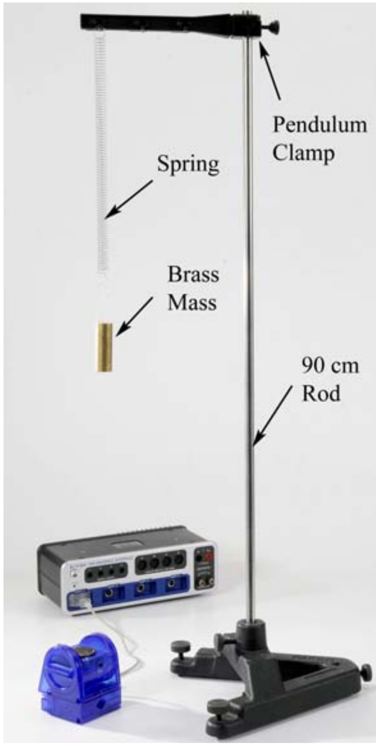

Figure 1. Simple Harmonic Motion

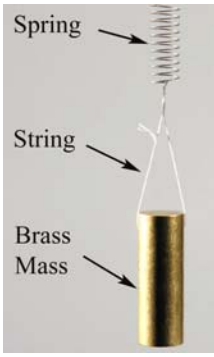

Figure 2. String

Procedure - Phase

1. Create the following calculations:
   v = smooth(15,derivative(5,[Position (m)],[Time (s)])) with units of m/s
   a = smooth(15,derivative(5,[v (m/s)],[Time (s)])) with units of m/s^2

2. On the graph, add a second vertical axis and put the calculation v on it. Change the units to cm/s.

3. Displace the mass from equilibrium by 10 to 20 centimeters, and allow it to oscillate. Take two or three periods worth of data. If the velocity data is not very smooth, try again. Get one good run of data.

4. Use the Coordinates tool to measure the period of the oscillation, T, and the time shift, ΔT, between the peak of position and velocity.

5. Calculate the ratio $\Delta T/T$. Is it about 1/4? This means that the phase of the velocity is 1/4 wavelength from the position.

6. A complete oscillation is also 360^o or 2π radians. What is 1/4 of that angle? How does this compare to the phase shown in Equation (2)?

7. Now add another vertical axis to the graph and put the calculation "a" on it. You can try smoothing this curve if it is not very sinusoidal. If it is too bad, take another run.

8. Use the Coordinates tool to measure the time shift, ΔT, between the peak of position and acceleration.

9. Calculate the phase shift between position and acceleration. How does this compare to the phase shown in Equation (3)?

10. Use a Sine curve fit on the position data to find the angular frequency, ω.

11. Record the amplitude, A, from the curve fit.

12. Use the Coordinates tool to measure the maximum velocity and maximum acceleration.

Analysis

1. Calculate v_{max} using Equation (4) and compare to your measured value.

2. Calculate a_{max} using Equation (5) and compare to your measured value.

Questions

1. What is the relationship between ω and the period you measured earlier?

2. Why did you need to only measure ω for the position graph?

3. When the position is at a positive maximum, the velocity is... negative maximum, zero, positive maximum?

4. When the position is at a positive maximum, the acceleration is... negative maximum, zero, positive maximum?

5. Can you answer questions 3 and 4 for when the position is at zero (equilibrium)? Why are there two answers?