16. Simple Harmonic Motion

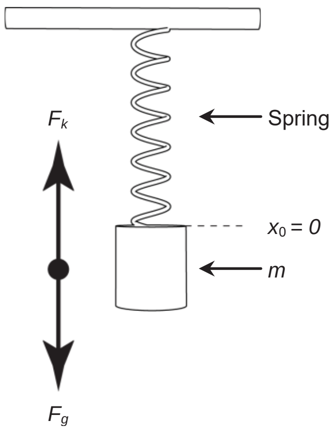
Driving Question

What factors influence the motion of an object that has a repeated or periodic motion?

Background

Simple harmonic motion generally describes the ideal motion of a body or system subject to a force proportional to the distance from some equilibrium position, that causes that body or system to move back and forth about that equilibrium position.

Consider a mass m, suspended from a spring.



When at rest, or in its equilibrium position x_0, the downward force of gravity on the mass is balanced by the upward force exerted by the stretched spring.

This upward force is given by:

$$F = -kx \qquad \textbf{Eq.1}$$

Where F is the force, **k** is the spring constant, and x is the distance the spring is stretched from its equilibrium position x_0. The negative sign in the equation implies that the force is a restoring force, pulling in the opposite direction as the force being exerted on it (in this case, the gravitational force on the attached mass). When the mass is pulled down a short distance (or moved up a short distance) and released, it will oscillate about its equilibrium position. Three forms of energy are present, kinetic energy, gravitational potential energy, and spring potential energy. In a perfect ideal system where there is no air resistance or internal friction, the sum of these three would be constant. Also, theory predicts that there is a relationship between the spring constant k, the period of oscillation T, and mass m:

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \textbf{Eq.2}$$

Materials and Equipment

For each student or group:

- ♦ Data collection system
- ♦ Force sensor
- ♦ Motion sensor
- ♦ Meter stick
- ♦ Rod stand
- ♦ Right-angle clamp
- ♦ Short rod
- ♦ Spring
- ♦ Assorted masses (at least 3)
- ♦ Balance (one per class)

Safety

Follow all standard laboratory procedures.

Sequencing Challenge

The steps below are part of the Procedure for this lab activity. They are not in the right order. Determine the proper order and write numbers in the circles that put the steps in the correct sequence.

◯	◯	◯	◯	◯
Raise the mass straight up, and then quickly remove your hand so that the mass begins to oscillate.	Stop data recording after several complete cycles have been recorded.	Connect the force sensor to the rod stand with the short rod and the right angle clamp.	Zero the force sensor	Compare the graph of Velocity versus Time, and the graph of Force versus Time

Procedure

After you complete a step (or answer a question), place a check mark in the box (☐) next to that step.

Note: When you see the symbol "�" with a superscripted number following a step, refer to the numbered Tech Tips listed in the Tech Tips appendix that corresponds to your PASCO data collection system. There you will find detailed technical instructions for performing that step. Your teacher will provide you with a copy of the instructions for these operations.

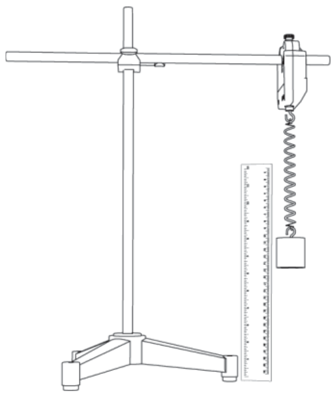
Part 1 – Determining the spring constant

Set Up

1. ☐ Start a new experiment on the data collection system. �(1.2)

2. ☐ Connect the force sensor to the data collection system. �(2.1)

3. ☐ Connect the force sensor to the rod stand using the short rod and the right-angle clamp.



4. ☐ Suspend the spring from the hook of the force sensor.

5. ☐ Push the "zero" button on the force sensor.

6. ☐ Put the data collection system into manual sampling mode with manually entered data. Name the manually entered data "Extension" and give it units of meters. �(5.2.1)

7. ☐ Place the meter stick near the spring with the scale oriented so that values increase in the downward direction.

8. ☐ Suspend the lightest mass you have selected for this activity from the end of the spring, and allow it to settle in its equilibrium position.

Collect Data

9. ☐ Start a manually sampled data set. �(6.3.1)

10. ☐ Starting with the first mass, collect a force value for each value of user-entered extension in meters (switching masses between each value you keep). �(6.3.2)

11. ☐ When you have a force and extension value recorded for each mass, stop data recording. �(6.3.3)

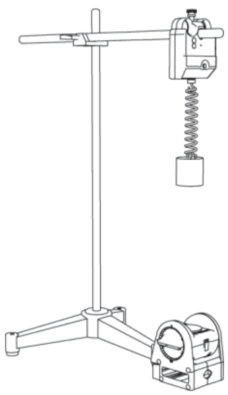
Analyze Data

12. ☐ Display Extension on the x-axis of a graph with Force on the y-axis. �(7.1.1)

13. ☐ Find the slope of a best fit line for your Force versus Extension data. The slope of the line is the spring constant k. Record that value in the Data Analysis section. �(9.6)

Part 2 – Mass oscillating in simple harmonic motion at the end of a spring

Set Up



14. ☐ Place the motion sensor below the mass. Depending on the materials you use, you may need to place the rod stand at the edge of a table and the motion sensor on the floor.

15. ☐ Ensure the motion sensor is set in the cart position. 

16. ☐ Select the smallest mass in the group you chose for this activity and hang the mass from the spring. Adjust the height of the force sensor so that the mass, at its lowest point while oscillating, is at least 15 cm above the motion sensor.

17. ☐ Change the sampling rate on the data collection system to 25 Hz. �(5.1)

18. ☐ Display both Force pull positive and Position on the y-axis of a graph with Time on the x-axis. �(7.1.10)

Collect Data

19. ☐ When the mass is motionless in its equilibrium position, push the “zero” button on the force sensor.

20. ☐ Carefully raise the mass a few centimeters, and then quickly remove your hand so that the mass begins to oscillate along a vertical line.

21. ☐ Start data recording, and then stop data recording after the mass has completed about twelve complete oscillations. �(6.2)

Analyze Data

22. ☐ Sketch a copy of your graphs in the Position versus Time and Force versus Time blank graph axes in the Data Analysis section.

23. ☐ Change the variable on the *y*-axis from Force to Acceleration. �(7.1.9)

24. ☐ Sketch a copy of your graphs in the Position versus Time and Acceleration versus Time blank graph axes in the Data Analysis section.

25. ☐ On the data collection system, change the variable on the *y*-axis from Acceleration to a Velocity. �(7.1.9)

26. ☐ Sketch a copy of your graphs in the Position versus Time and Velocity versus Time blank graph axes in the Data Analysis section.

27. ☐ Using your graph of Position versus Time and the data analysis tools on the data collection system, determine the time it took the spring and mass system to complete ten oscillations, and then record that value in Table 1 in the Data Analysis section. �(9.2)

28. ☐ Using your graph of Position versus Time and the data analysis tools on the data collection system, determine the amplitude of oscillation for the system, and then record that value in Table 1 in the Data Analysis section. �(9.2)

Note: the amplitude is the displacement from the equilibrium position, so you will have to calculate half the distance from a maximum peak to a minimum peak.

29. ☐ Record data for each of the masses you have, and find the time it takes to complete 10 oscillations and the amplitude of oscillation for each mass.

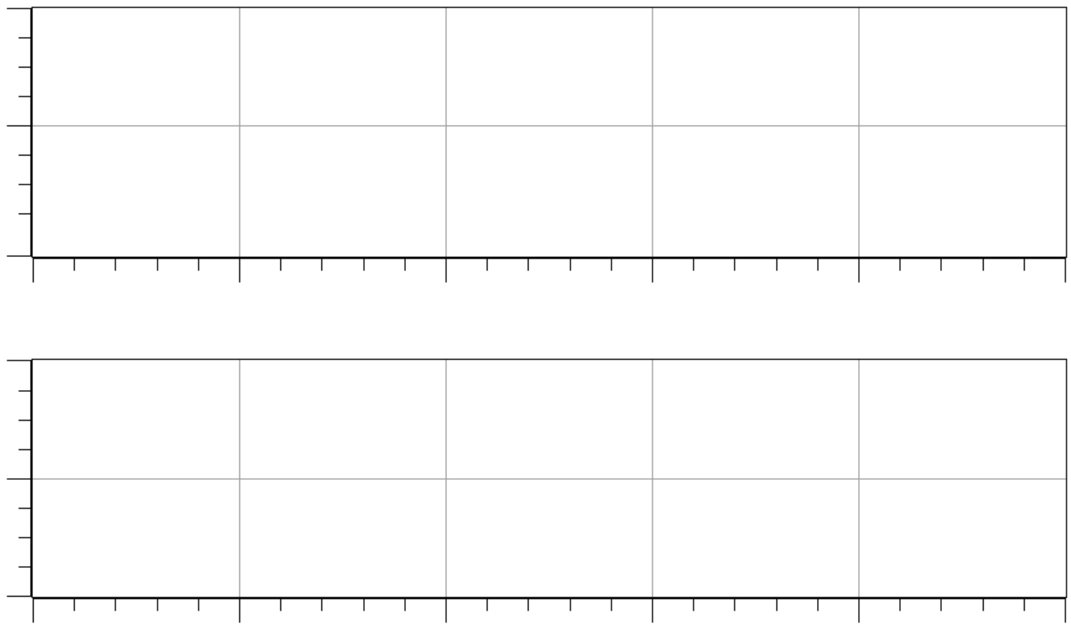
30. ☐ Complete Table 1 in the Data Collection section.

31. ☐ Save your experiment as instructed by your teacher. �(11.1)

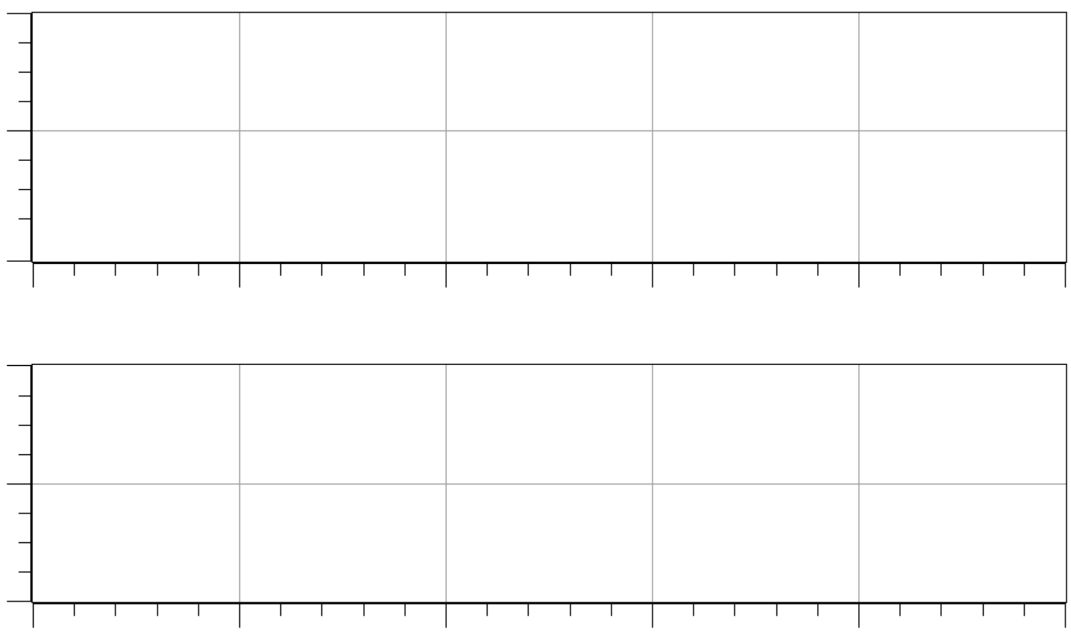
Data Analysis

$k =$ ____________________

Force versus Time and Position versus Time



Acceleration versus Time and Position versus Time



Velocity versus Time and Position versus Time

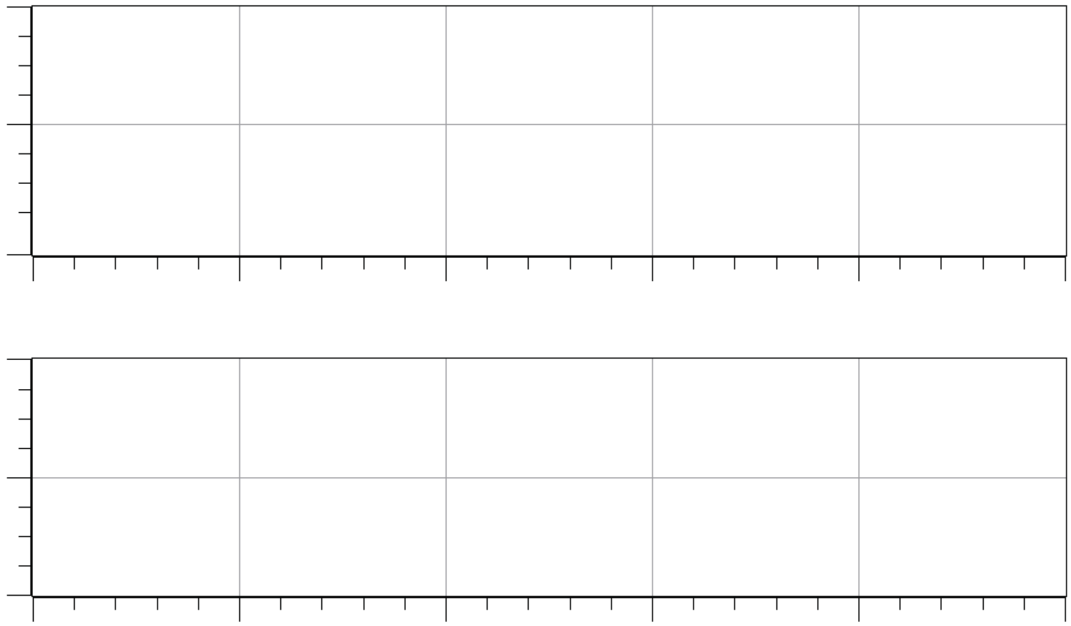


Table 1: Mass and period – measured

Mass (kg)	Time for 10 oscillations (s)	Period of one oscillation (s)	Frequency of oscillation (Hz)	Amplitude of oscillation (m)

1. ☐ The expression shown below shows the theoretical relationship between the spring constant *k*, the mass *m*, and the period of oscillation, *T*.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

For each of the masses used, calculate the theoretical period of oscillation and enter the values into Table 2. Then, calculate the percent error in the measured period values for each mass. Enter the percent error values into Table 2.

Table 2: Mass and period – theoretical

Mass (kg)	*k* (N/m)	Theoretical Period (s)	% Error

2. ☐ The theoretical relationship assumes that the mass of the spring is zero. Depending on the mass and spring you are using, the size of this systemic error will vary. To correct for this error we will assume that the additional mass added to the system is approximately equal to 1/3 the mass of the spring.

3. ☐ Use a balance to measure the mass of the spring, and then record the value below.

m_{spring} = ____________________

4. ☐ Add one third of the mass of the spring to the total mass m used in your theoretical calculations, and then re-calculate new, corrected, theoretical values for the period of oscillation for each mass as well as the percent error associated with your measured values. Record the new mass values and the results of your calculations into Table 3.

Table 3: Mass and period

Mass + 1/3 Spring (kg)	k (N/m)	Theoretical Period (s)	% error

Analysis Questions

1. To determine the period of oscillation for the spring, why is it better to measure the time for 10 oscillations and then divide the value by 10 than it is to simply measure the time for one oscillation?

2. Why was the sampling rate increased to 25 Hz for the experiment?

3. Which of the motion graphs that resulted from measurements made by the motion sensor is most closely related to the graph of Force versus Time?

4. Describe the relationship between the Position versus Time, Velocity versus Time, and Acceleration versus Time graphs for the oscillating mass.

5. All three variables (position, velocity, and acceleration) are constantly changing; but because the motion is repeated over and over, there are some definite relationships.

a. What is the value of the acceleration when the velocity has its greatest magnitude? When does this occur during the cycle of one oscillation?

b. What is the value of the velocity when the acceleration has its greatest magnitude? When does this occur during the cycle of one oscillation?

6. **How did the addition of one third the spring mass affect the percent error in your measured values? How would you spot or verify a systematic error like this?**

Synthesis Questions

Use available resources to help you answer the following questions.

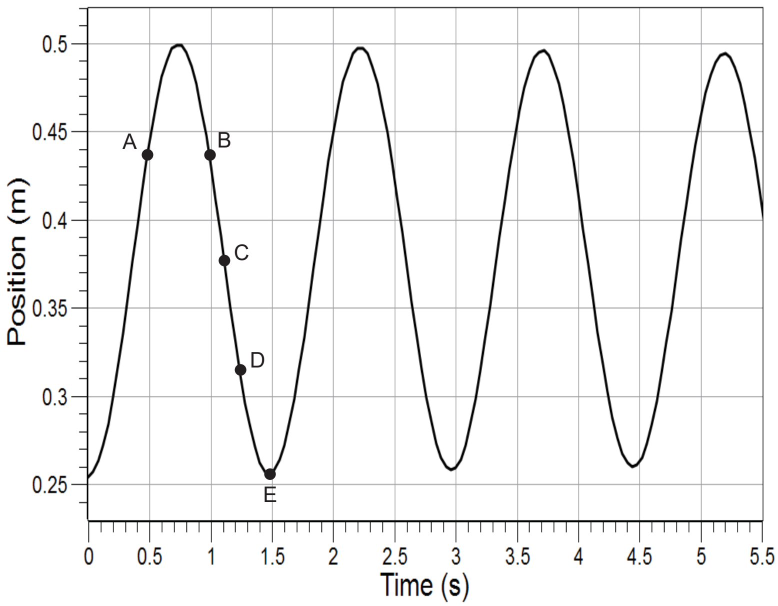
1. **If this activity were repeated on the surface of the moon using the same equipment, would the results be the same? Explain.**

2. **If this activity were repeated by astronauts on a space shuttle using the same equipment, would the results be the same? Explain.**

3. **Springs play an important role in the suspension systems of automobiles to give a comfortable ride. However, these suspension systems also include shock absorbers. What is their role?**

Multiple Choice Questions

Select the best answer or completion to each of the questions or incomplete statements below.



The graph above shows the position of a mass as it oscillates vertically on the end of a spring in simple harmonic motion. Five points show various positions of the mass during oscillation. Assume that up is positive and down is negative.

1. **At which position is the mass's acceleration equal to zero?**

- **A.** A
- **B.** B
- **C.** C
- **D.** D
- **E.** E

2. **In which two positions is the mass's acceleration negative, or downward?**

- **A.** A & B
- **B.** B & D
- **C.** D & E
- **D.** A & E
- **E.** B & E

3. **In which two positions is the net force positive, or upward?**

- **A.** A & B
- **B.** B & C
- **C.** C & D
- **D.** D & E
- **E.** A & E

4. **At which position is the net force zero?**

- **A.** A
- **B.** B
- **C.** C
- **D.** D
- **E.** E

5. **At which position is the magnitude of the mass's velocity the greatest (when is the mass moving the fastest)?**

- **A.** A
- **B.** B
- **C.** C
- **D.** D
- **E.** E

6. **Which point represents the greatest displacement from the rest position?**

- **A.** A
- **B.** B
- **C.** C
- **D.** D
- **E.** E

7. **Which of the following would increase the period of oscillation?**

- **A.** Increase the mass
- **B.** Decrease the mass
- **C.** Use a spring with a smaller k value
- **D.** Both A and C are correct
- **E.** Both B and C are correct

8. **In which position are velocity and acceleration in the same direction?**

- **A.** A
- **B.** B
- **C.** C
- **D.** D
- **E.** E

Key Term Challenge

Fill in the blanks from the list of randomly ordered words in the Key Term Challenge Word Bank.

1. For an oscillating mass and spring system, a graph of net force acting on the mass versus time will be in phase with a graph of ________________ versus time for the same mass. The two factors that affect the period of oscillation of a mass and spring system are the magnitude of ________________ and the ________________ of the spring. If the mass in an oscillating mass and spring system were increased to four times its original mass, the period of oscillation would change by a factor of ________________.

Key Term Challenge Word Bank

Paragraph 1

Acceleration

Mass

Spring constant

Two

Four

Eight