External Force and Newton's Laws

Introduction

Consider the system of the two Smart Carts connected by a string, and being pulled by an elastic band, as shown in Figure 1. The external force exerted by the hand through the elastic band causes the system to accelerate. The carts' sensors are used to measure the forces and the acceleration. Theoretical values of the acceleration and string tension are calculated using Newton's laws, and then are compared to the measured values.

Figure 1: System Being Pulled By Elastic Band

Equipment

Qty	Items	Part Number
1	Smart Cart (Red; including Hook)	ME-1240
1	Smart Cart (Blue; including Hook)	ME-1241
1	Dynamics Track Feet (Pair)	ME-8972
1	Dynamics Track End Stops (Pair)	ME-8971
1	1.2m Starter Dynamics Track	ME-9493
1	Elastic Bumper (Only elastic band needed)	ME-8998
1	String	SE-8050
Required, but not included:		
1	Balance	SE-8723
1	PASCO Capstone Software	

Theory

As illustrated in Figure 2, an external force F applied to the system causes acceleration "a". The two masses, M_1 (red cart) and M_2 (blue cart), are connected by a string with tension T, which is a force internal to this system. Note: This neglects the small frictional forces, and ignores the vertical forces which cancel out.

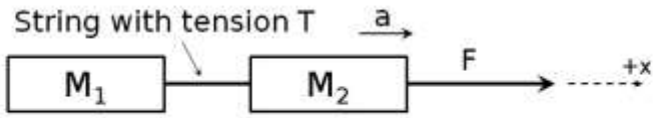

Figure 2: System of Two Carts and String

Applying Newton's 2nd Law to the system, along the x-axis, yields

$$F = (M_1 + M_2)a \qquad (1)$$

The tension is internal, and so does not appear in this equation, which includes only external forces.

The two masses can also be treated as separate systems.

Figure 3 shows only M_1 (red cart), with the string tension T as an external force. Applying Newton's 2^{nd} Law here gives

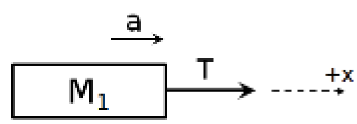

Figure 3: System of Red Cart Only

$$T = M_1 a \qquad (2)$$

Figure 4 shows only M_2 (blue cart), with both F (to the right) and T (to the left, so this has a negative component along the x-axis) as external forces. Applying Newton's 2^{nd} Law in this case yields

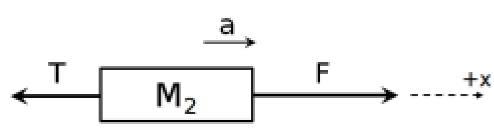

Figure 4: System of Blue Cart Only

$$F - T = M_2 a \qquad (3)$$

1. Show that Equations 2 and 3 combine to give Equation 1.

2. Show that Equations 1 and 2 combine to give Equation 4:

$$T = M_1 F / (M_1 + M_2) \qquad (4)$$

Setup

1. As shown in Figure 5, set up the track with feet underneath at both ends. On the upper side of the track, attach an end stop at each end.

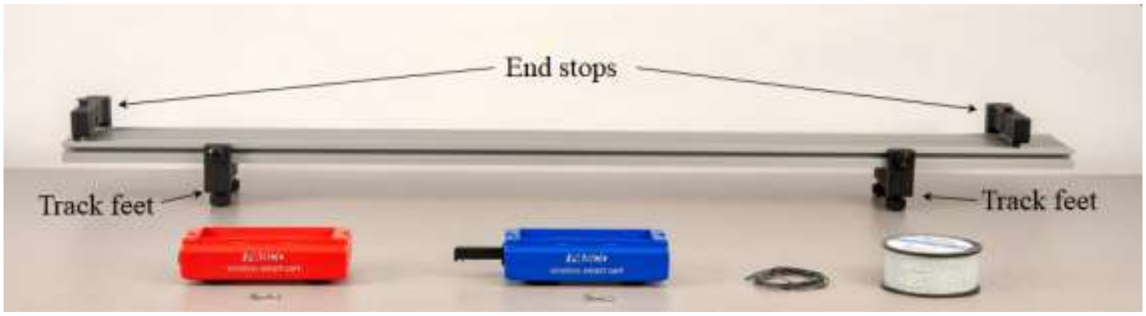

Figure 5: Experimental Setup

2. Adjust the feet to make the track level. To test the level: Place a cart on the track and give the cart a little push in one direction to see if it coasts to a stop or accelerates. Then push it in the opposite direction to see if the cart coasts to a stop equally in both directions.

3. To set up the carts as shown in Figure 6:

   Attach a Hook to each Smart Cart. Each hook is now attached to a Smart Cart Force Sensor.

   Push the button on top of the Blue cart to release its plunger. Tie a short string to the plunger. At the other end of the string, tie a loop and connect the loop to the hook on the Red cart. Tie a piece of Elastic Band to the hook on the Blue cart, leaving the band long enough that you can pull on it horizontally.

   Pull horizontally on the Elastic Band, and make sure the string connecting the carts is also horizontal when under tension. If not, adjust the string as needed.

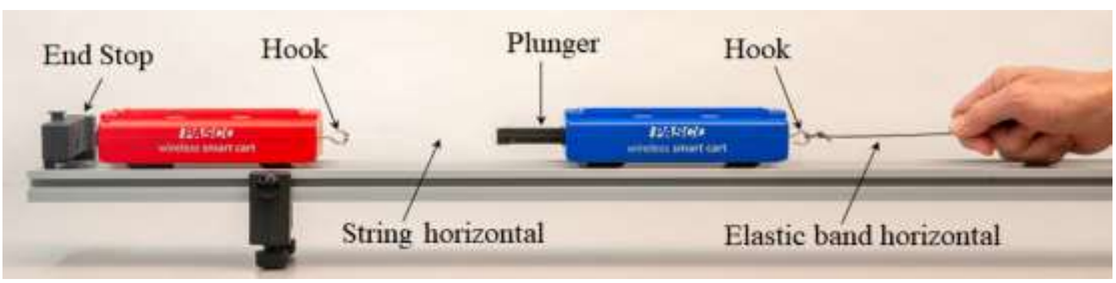

Figure 6: Setup Just Before Acceleration

In this arrangement, the Blue Smart Cart Force Sensor will measure the external force F you apply to the system through the elastic band. The Red Smart Cart Force Sensor will measure the tension T in the string, and one of the position sensors will be used to determine the acceleration "a" of both carts.

4. Turn on the Smart Carts, and then wirelessly connect them via Bluetooth in Capstone software.

5. In the lower toolbar, choose the Smart Cart Position Sensor for the Red cart and set the sample rate to 40 Hz. Repeat for the Red cart's force sensor. Then repeat for the position and force sensors of the Blue cart.

6. Create a graph of Force vs. Time for the Blue cart force sensor. Add a similar measurement to the vertical axis and choose Force for the Red cart.

7. Add a new plot area to the graph, and select Acceleration (under Smart Cart Position Sensor for one of the carts). This acceleration is calculated from the position sensor data. (Do not select data from the Smart Cart Acceleration Sensor.)

8. To zero a Smart Cart's force sensor:

   Place the cart at rest on the track with no tension in the string or elastic band, so there is no force pulling on either hook. In the lower toolbar in Capstone: Select the Smart Cart Force Sensor for that cart (Red or Blue), and then Zero the force sensor.

   Repeat for the other cart.

   To test: Start recording. The force graph should read nearly zero since the string is removed. If not nearly zero, then stop recording, re-zero, and restart recording.

9. Recording conditions: In Capstone, on the lower toolbar, open Recording Conditions. Create a Start Condition: Position is above 0.10. Create a Stop Condition: Position is above 0.40. With these settings, data recording begins only when the carts have traveled 0.10 m (by which point the acceleration will be steady, making for a nicer graph), and ends at 0.40 m. You can adjust these Recording Conditions as needed to suit your experiment.

Procedure

1. Back the carts up so the Red cart is at one of the end stops, as in Figure 6. Without recording any data, practice pulling on the carts with the elastic band. Just before accelerating the carts, hold the elastic band with a very small amount of tension, so that there is also a very small tension in the string connecting the carts (see Figure 6). Then pull harder, to accelerate the carts, keeping tension in the string as far as possible along the track. Make sure you pull only horizontally.

2. Start recording. Then pull on the elastic band, accelerating the carts. Recording on the graph should automatically start and stop, according to the recording conditions. Repeat as needed until you get one good run of data, where you pull only horizontally until the recording automatically stops. It is fine if the force increases or decreases throughout the run. Adjust recording conditions if needed.

3. Open the Data Summary where you can change the color of each trace, so that the Red Cart Force Sensor data is red and the Blue Cart Force Sensor data is blue. This makes it easier to keep track.

4. Use a balance to measure each cart mass and record.

Analysis

1. Examine the two curves on the force graph to see if the blue and red carts' forces are the same. Should they be?

2. Pick a point in time and measure both forces and the acceleration using the Multi-Coordinates tool from the graph tool palette. For example, the peak force makes a convenient location, but you can pick any point where you have good data. Record your values.

3. Use the measured values of M_1, M_2, and F in Equation (1) to calculate the theoretical acceleration a.

4. Use the measured values of M_1, M_2, and F in Equation (4) to calculate the theoretical Tension T.

5. Compare the measured value of a to the theoretical value using the percent error calculation:

$$\%\ \text{Error} = \frac{\text{Measured} - \text{Theoretical}}{\text{Theoretical}} \cdot 100$$

6. Repeat to compare the measured value of T to the theoretical value.

7. Were your measured values high or low compared to the theoretical values? What might account for this?