Lab 21: Four-Point Bend Testing

Introduction

A Four-Point Bend Test is performed on plastic beams as shown in Figure 1. As a downward force (F) is applied in the middle of the beam, the flex (Δx) is recorded. The ratio (F/Δx) is the effective stiffness of the length of beam being tested, and is measured directly from the slope of the F vs. Δx graph. The Flexural Elastic Modulus for the material is then calculated.

This experiment uses the ABS plastic beams from the PASCO Structures System. You will need to cut each beam to a length of 10 cm, so that it will fit between the drive screws.

Equipment

<table>
<thead>
<tr>
<th>Qty</th>
<th>Items</th>
<th>Part #</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Materials Testing Machine</td>
<td>ME-8236</td>
</tr>
<tr>
<td>1</td>
<td>Bonding Accessory</td>
<td>ME-8227</td>
</tr>
<tr>
<td>1</td>
<td>Structures Flat Beam</td>
<td>ME-6987</td>
</tr>
<tr>
<td>1</td>
<td>Four-Point Load Anvil</td>
<td>ME-8249</td>
</tr>
<tr>
<td>1</td>
<td>Calipers</td>
<td>SE-8710</td>
</tr>
</tbody>
</table>

Note: The ME-8237 Bending Accessory consists of the upper (single point) load anvil and the lower base with the two support anvils. In this lab, you will use the ME-8249 Four-Point Load Anvil instead of the single point anvil supplied with the ME-8237.
Four-Point Bend Test

A test sample is supported by two anvils separated by a length "L", as shown in Figure 2. A load is applied by two load anvils, each set an equal distance "a" from the lower load anvils. These distances are measured from the anvils, NOT the end of the beam. A total load "F" is applied (½F by each load anvil) to the test sample, and the resulting anvil deflection "Δx" is measured. The ratio "F/Δx" is the stiffness of the sample, and depends on the length "L" and the load anvil spacing "L_o". It also depends on the shape and area of the sample cross-section, as well as the material.

If "E" is the Flexural Elastic Modulus for the material, and "I" is the Area Moment of Inertia for the sample, then

\[ \frac{F}{\Delta x} = \frac{12EI}{a^2(3l - 4a)} \]

Eqn. (1)

Since \( L = 2a + L_o \), the distance "a" can be eliminated, and Eqn. (1) written as

\[ \frac{F}{\Delta x} = \frac{48EI}{(L+L_o)(L-L_o)^3} \]

Eqn. (2)

The spacing for the anvils could be any amount; but two common used standards are \( L = 2L_o \) and \( L = 3L_o \). For these two special cases, Eqn. (2) becomes

for \( L = 2L_o \),

\[ \frac{F}{\Delta x} = \frac{16EI}{(L_o)^3} \]

Eqn. (3)

and for \( L = 3L_o \),

\[ \frac{F}{\Delta x} = \frac{3EI}{(L_o)^3} \]

Eqn. (4)

Note: "\( \Delta x \)" is the deflection of the load anvils, NOT the maximum deflection of the test sample at its center.

The Area Moment of Inertia depends on the cross-sectional shape and area of the sample. For a beam with a rectangular cross-section

\[ I_{\text{rectangle}} = \frac{1}{12} A h^2 \]

Eqn. (5)

where "A" is the cross-sectional area, and the height "h" is the dimension that is parallel to the applied force. The base "b" is the dimension perpendicular to the applied force, and since \( A = bh \), Eqn. (5) can be written as

\[ I_{\text{rectangle}} = \frac{1}{12} b h^3 \]

Eqn. (6)
Setup

1. Connect the Four-Point Load Anvil to the cross-head using the knurled cap nut as shown in Figure 1.

2. The base (for the support anvils) fastens directly to the load cell using the two cap screws as shown in Figure 3.

3. Carefully measure the spacing, $L_o$, of the upper fixed anvil. This measurement is from the top of the camber on each anvil, or you can measure between the vertical surfaces, and calculate $L_o$ by including the 1.5 mm radius on each anvil. Record this value below.

4. Adjust the spacing "L" of the lower support anvils so that $L=2L_o$. Make sure that the upper anvil is centered between the two lower anvils. Use calipers to make this alignment as accurate as possible.

5. Cut a 10 cm length of rectangular cross-section beam from the ME-6987 Structures Flat Beam set. You can use either the F4 or 3X4 beams from that set. The shorter 2X3 beam (also in that set) has a smaller cross-section and should be saved for further investigations, later.

6. Measure the cross-sectional dimensions of the beam and record.

7. Use Eqn. (6) to calculate the Area Moment of Inertia for the beam and record.

8. Place the beam across the support anvils as shown in Figure 3. Turn the crank counter clockwise until the load anvil is just touching the sample.

\[
\begin{align*}
L_o &= 3.0 \text{ cm} \\
\text{Cross-section} &= 2.61 \text{ mm} \times 10.18 \text{ mm} \\
I &= \left(\frac{1}{12}\right) b h^3 \\
&= \left(\frac{1}{12}\right) \left(0.00261 \text{ m}\right) \left(0.01018 \text{ m}\right)^3 \\
&= 2.295 \times 10^{-10} \text{ m}^4
\end{align*}
\]
**Taking Data**

Note: The sample rate is set to 5 Hz, but you can change this if needed. In general, a slower rate gives smoother (less noisy) data.

1. Click on Record. Turn the crank counter-clockwise, bending the sample. Increase the force to about 300 N.

2. Click on Stop. The data should be fairly linear. It is OK if there is a slight curvature at the beginning or end, but if there is not a straight section in the middle, you probably have something wrong.

3. Use a linear curve fit to find the slope. This is the stiffness ($F/\Delta x$) of the length of beam you are testing. Take multiple runs to get a good average value.

\[
\text{slope} = \frac{F}{\Delta x} = 298000 \text{ N/m}
\]
Analysis

1. Use Eqn. (4) to calculate “E”, the Flexural Elastic Modulus. Estimate the uncertainty in your value for E.

2. How does the Flexural Modulus compare to the value found in reference data tables for Young’s Modulus for ABS plastic?

For Further Study

1. Bend the sample in the weak direction. Does this affect the value for E?

2. Try different spacing for the anvils. L = 3L₀. Does this affect the value for E?

3. Try a smaller cross-section. Use the 2x3 beam from the ME-6987 Structures Flat Beam set.

4. Try a different shape, such as the ME-7012 Thin I-beam

5. Try a different material, such as the ME-7011 Polycarbonate Beams or the ME-6983 Carl Beams.

Answers:

1. \[ E = \frac{(F/A)(I_0)^3}{16T} \]
   \[ = \frac{(296000)(.030)^3}{16(2.295\times10^{-10})} = 2.2 \pm .1 \text{ GPa} \]

2. Reference tables give E = 2.3 GPa for ABS.