Lab 13: Column Buckling and Slenderness Ratio

Introduction

Three different length plastic I-beams are tested under compression to investigate their method of failure. The way in which a member fails (buckling or not) is determined by its Slenderness Ratio, and this ratio is calculated for each beam. Topics covered also include the Radius of Gyration and the Area Moment of Inertia.

This experiment uses the #2, #3 and #4 ABS plastic beams from the PASCO Structures System.

Equipment

<table>
<thead>
<tr>
<th>Qty</th>
<th>Items</th>
<th>Part #</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Materials Testing Machine</td>
<td>ME-8236</td>
</tr>
<tr>
<td>1</td>
<td>Structures Beam Adapter</td>
<td>ME-8242</td>
</tr>
<tr>
<td>1</td>
<td>Structures Truss Set</td>
<td>ME-6993</td>
</tr>
<tr>
<td>1</td>
<td>Calipers</td>
<td>SE-8710</td>
</tr>
</tbody>
</table>

Written by Jon Hanks

Setup

1. The ME-8242 Structures Beam Adapter consists of two major parts: the upper fixture with the longer thread sticks up through the cross-head and is held in place by the knurled cap nut.

2. The lower fixture screws directly into the Load Cell as shown in the inset to Figure 1.

3. Install the shortest beam (#2) and secure the covers on both fixtures using the cap screws.
Theory of Columns

If a very short bar is compressed, it will shorten. If it is compressed even more, past the elastic limit of the material, it bulges until it is deformed into a flat disk. However, if the bar being compressed is sufficiently long (and straight), something different can happen. At first, as the load is increased, the bar shortens as before. But as the force increases, a critical value is reached, and the beam suddenly buckles. In general, if a compression member is long enough to fail by buckling, it is called a column; otherwise it is simply a compression member.

For a given material, the way in which a member fails (buckling or not) is determined by its Slenderness Ratio; the length (L) of the member divided by the Radius of Gyration (k) of the members cross-sectional area.

\[ k = \sqrt{I/A} \quad \text{Eqn. (1)} \]

where \( I \) is the Moment of Inertia, and \( A \) is the area of the cross-section.

For a beam member with a rectangular cross section

\[ I_{\text{rectangle}} = \frac{1}{12} A \ h^3 \quad \text{Eqn. (2)} \]

where the height, \( h \), is the dimension that is parallel to the direction in which the beam will buckle. The base, \( b \), is the dimension perpendicular to this direction (see Fig. 2), and since \( A = bh \), Eqn. (2) can be written as

\[ I_{\text{rectangle}} = \frac{1}{12} b \ h^3 \quad \text{Eqn. (3)} \]

Calculating Radius of Gyration, \( k \).

1. Measure the cross-sectional dimensions of your beam, and use Eqn. (3) to calculate the Moment of Inertia. You can assume that the cross-section is composed of three rectangles, and that all the beams are the same.

2. Calculate the cross-sectional area of the beam.

3. Use Eqn. (1) to calculate Radius of Gyration of the beam.

Flange: thickness = 1.55 mm width = 10.15 mm
Web: thickness = 2.50 mm width = 6.80 mm

\[ I = \left( \frac{1}{12} \right) (.0068) (.0025)^3 + 2(\frac{1}{12}) (.00155) (.01015)^3 \]
\[ = 2.79 \times 10^{-10} \text{ m}^4 \]

\[ A = (.0068) (.0025) + 2 (.00155) (.01015) = 4.85 \times 10^{-5} \text{ m}^2 \]

\[ k = (2.79 \times 10^{-10} / 4.85 \times 10^{-5})^{.5} = 2.4 \times 10^{-3} \text{ m} \]
Taking Data

1. With the #2 beam installed, measure the distance between the top of the lower fixture to the bottom of the upper fixture. This is the effective length, \( L \), of the beam. The amount that is free to bend. Record in the table below.

2. Click on Record and turn the crank counterclockwise, compressing the beam. Continue until the beam fails, and click on Stop.

3. Observe the manner in which the beam fails. Does it buckle in the “weaker” direction for the beam? Note that because of the holes in the beam, it will fail no matter what its length is. But does it fail in compression along the side of the hole, or does it suddenly buckle sideways and fail as a column?

4. Repeat the above procedure for the #3 and then #4 beams.

<table>
<thead>
<tr>
<th>Beam</th>
<th>( L ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.031</td>
</tr>
<tr>
<td>2</td>
<td>0.068</td>
</tr>
<tr>
<td>3</td>
<td>0.120</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

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Analysis

1. For each beam, calculate the Slenderness Ratio, the effective length (L) of the member divided by its Radius of Gyration (k).

   \[
   \text{Slenderness Ratio} = \frac{L}{k}
   \]

2. Record the value in the table below.

3. What was the mode of failure for each beam? Which beam is truly a column?

Answers:

The #2 length beam never buckles and the #4 length always buckles. The #3 length can fail either way.

<table>
<thead>
<tr>
<th>Beam</th>
<th>L (m)</th>
<th>Slenderness Ratio</th>
<th>Buckle?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>#2</td>
<td>0.031</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>#3</td>
<td>0.068</td>
<td>28</td>
</tr>
<tr>
<td>3</td>
<td>#4</td>
<td>0.120</td>
<td>50</td>
</tr>
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